Minimum noise trajectories for ground takeoff to 5000-ft terminal altitude were also generated for both the constrained noise and unconstrained noise cases. Again, maximum sustainable flight path angle is achieved in steady state.

However, due to the constraint on instantaneous annoyance, the maneuvers are quite severe and unrealistic. Large thrust excursions occur when the noise is constrained to 95 PNdb at both microphone one (0.5 naut mile downrange) and microphone two (1 naut mile downrange). In fact, if a microphone were placed 0.75 naut mile from the runway (in addition to the 0.5-naut mile and 1.0-naut mile locations), the instantaneous noise level constraint of 95 PNdB would be violated. This presents a case for concentrating more microphones at the beginning of the ground track so that if the noise level constraint can be satisfied it will be satisfied more uniformly along the ground track, and hopefully a smoother trajectory would be generated.

To compare the effect of microphone position, the unconstrained ground takeoff case was repeated with all microphones shifted 0.5 naut mile downrange. Steady-state conditions were the same in both cases, but the time to reach steady state was less when the first microphone was downrange 1.0 naut mile instead of 0.5 naut mile.

IV. Conclusions

A steepest descent optimization program is used to determine minimum annoyance takeoff and climbout trajectories for a STOL vehicle. Although the vehicle model and annoyance function is quite sophisticated compared with previous work, improvements may be made by including in the annoyance function directional noise and disturbing high frequency noise components. However, until better noise field prediction techniques are developed, the results here are felt to give important trends. From various initial conditions, the trajectory begins with an initial transient which quickly settles into a steady state of maximum sustained flight path angle at full thrust. In the case of bounded maximum noise, the thrust drops radically during the initial transient. These maneuvers are unrealistic and might be softened with a slight increase the noise bound. For some choices of initial conditions, the angle-of-attack bound may be reached. This and the noise bound, which are functionally dependent upon both the state and control variables, are handled with only a little additional complexity in the steepest descent problem.[5] Thus, a far more sophisticated model is investigated without the need of the computationally cumbersome dynamic programming.

References

[1]DeMaio D. A., "Optimum Noise Abatement Trajectories," M..I.T., Unpublished Memo, March 1971, Charles Stark Draper Laboratory, Cambridge, Mass.

[2]Erzberger, H. and Lee, H. Q., "Technique for Calculating Optimum Takeoff and Climbout Trajectories for Noise Abatement," TN D-5182, May 1969, NASA.

[3]Hays, A. P., "Noise Minimization of Helicopter Take-off and Climb-out Flight Paths Using Dynamic Programming," MIT, Cambridge, Mass. M. S. thesis, May 1971, Dept. of Aeronautics and Astronautics, M.I.T.

[4]Bryson, A. E., Jr. and Ho, Y. C., *Applied Optimal Control*, Blaisdell, Waltham, Mass. 1969.

[5]Sperry, W. C., "Aircraft Noise Evaluation," TR FAA-no-68-34, Sept. 1968, FAA Office of Noise Abatement.

[6]Lee, R. et al, "Procedures for Estimating the Effects of Design and Operational Characteristics of Jet Aircraft on Ground Noise," CP-1053, June 1963, NASA.

[7]Boeing Company, "Study and Development of Turbofan Nacelle Modifications to Minimize Fan-Compressor Noise Radiation," Vol. 1, NASA CR-1711, Jan. 1971.

[8]Kramer, J. J., "Quick Engine Program, Detailed Engine Designs," Paper #19 in Progress of NASA Research Relating to Noise Alleviation of Large Subsonic Jet Aircraft, NASA SP-189, 1968.

[9]Hildebrand, F. B., *Introduction to Numerical Analysis*, McGraw-Hill, New York, 1956, pp. 60–64.

[10]Zeldin, S., and Speyer, J., "Maximum Noise Abatement Trajectories," AIAA Paper 72-665, 1972, Boston, Mass.

[11]Denham, W. F. and Bryson, A. E., Jr., "The Solution of Optional Programming Problems with Inequality Constraints," *AIAA Journal*, Vol. 1, No. 11, Nov. 1965, pp. 2544–2500.

On the Inverse Calculation of the Mass Flow Parameter

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On the Inverse Calculation of the Mass Flow Parameter

THE mass flow parameters for one-dimensional internal flow are related to the Mach number as

$$\dot{m}_s = M\left[\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{1/2} \qquad (1)$$

$$\dot{m}_t = M\left[\frac{\gamma g}{R}\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{\gamma-1}}\right]^{1/2} \qquad (2)$$

where g is gravitation acceleration, R is the universal gas constant, and γ is the specific heat ratio. The mass flow parameters in terms of static and total pressures respectively are given as

$$\dot{m}_s = [W(T_t)^{1/2}/P_s A]; \qquad \dot{m}_t = [W(T_t)^{1/2}/P_t A]$$

Here W = weight flow rate, T_t = total temperature, P_s,P_t = static and total pressures respectively, and A = cross sectional flow area. The functional relationships between Mach number and m_s and m_t for air (γ = 1.4) are shown in Fig. 1.

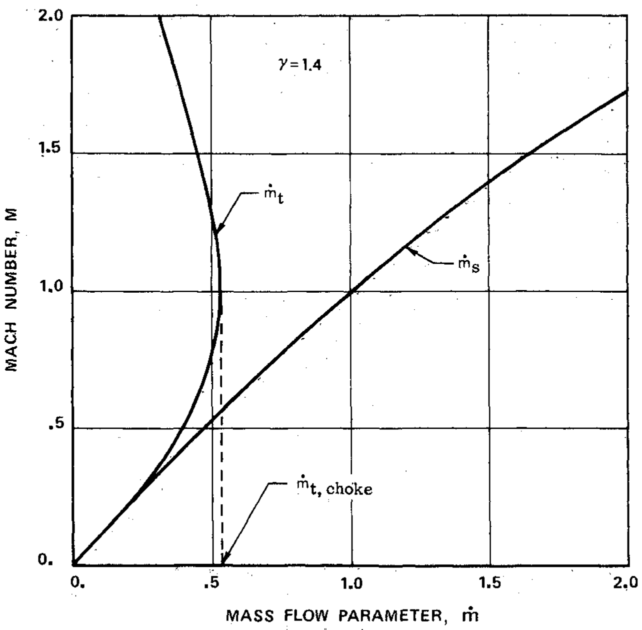

Fig. 1 Mach number variation with total pressure and static pressure mass flow parameters.

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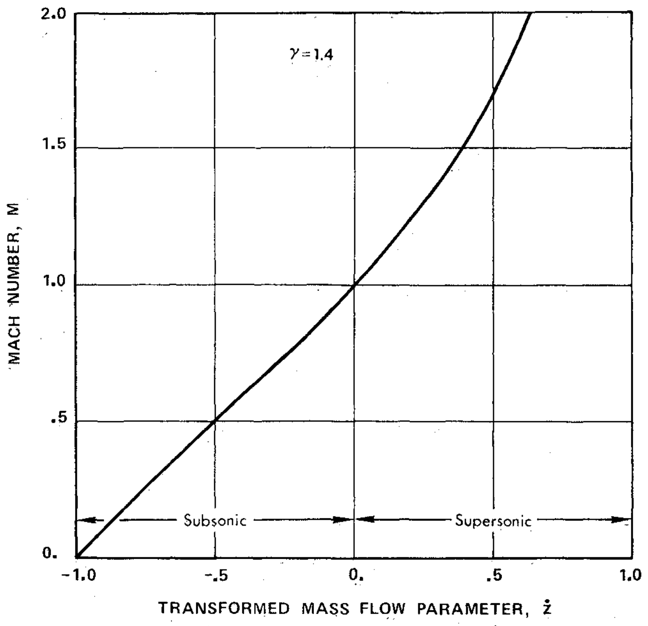

Fig. 2 Mach number variation with transformed total pressure mass flow parameter.

In many situations, however, $\dot{m}_s$ or $\dot{m}_t$ is known, and one needs to determine the Mach number. Equation (1) has a closed form inverse which can be written as

$$M^2 = -\frac{1}{\gamma - 1} + \left[\left(\frac{1}{\gamma - 1}\right)^2 + \frac{2R\dot{m}_s^2}{\gamma g(\gamma - 1)}\right]^{1/2} \tag{3}$$

The equation has another root that corresponds to a negative M^2 which is physically unreal. A closed form solution for Eq. (2), however, is formidable. One usually resorts to the interpolation of tables, curves, or for machine computations, iterative schemes. Due to the infinite slope of the Mach number with respect to the mass flow parameter at the sonic point, however, one usually encounters the problem of numerical singularity. That is, the solution is difficult to obtain near $M = 1$. Using the Newton method of iteration, for example, one can reliably obtain a solution only up to a Mach number of about 0.9 in the subsonic region.

It has been found that such numerical singularity can be overcome by rewriting Eq. (2) in terms of a simple transformed mass flow parameter which is defined as

$$\dot{z} = \begin{cases} +\left(1 - \dfrac{\dot{m}_t}{\dot{m}_{t,\,\text{choke}}}\right)^{1/2} & M > 1.0 \\ -\left(1 - \dfrac{\dot{m}_t}{\dot{m}_{t,\,\text{choke}}}\right)^{1/2} & M < 1.0 \end{cases} \tag{4}$$

where $\dot{m}_{(t)\text{choke}}$ is the mass flow parameter at the sonic or choking condition. Thus, Eq. (2) becomes

$$\dot{z} = \pm\left\{1 - M\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{-\frac{\gamma+1}{2(\gamma-1)}}\right\}^{1/2} \tag{5}$$

Where (+) and (−) are for supersonic and subsonic flows, respectively. The Mach number variation with respect to the transformed mass flow parameter $\dot{z}$ for air is plotted in Fig. 2. The transformed relationship has two distinctive advantages: 1) The infinite slope at $M = 1$ has disappeared, thus, eliminating numerical problems near $M = 1$. 2) The curve is nearly linear in the subsonic region, and thus one may closely approximate the Mach number by the relation

$$M \approx 1 - \dot{z}^2 \qquad 0 < M < 1.0 \tag{6}$$

In fact, Eq. (6) is exact for $\gamma = 1$. Thus, in any iterative scheme, Eq. (6) can be used as a close first approximation.

The inverse solution of Eq. (5) is ideal for the Newton iterative scheme, where successive approximations to the Mach numbers are given by

$$M_{i+1} = M_i - (\dot{z}_i - \dot{z})\left(\frac{dM}{d\dot{z}}\right)_i \tag{7}$$

where

$$\left(\frac{dM}{d\dot{z}}\right)_i = \frac{2\dot{z}_i\left[\dfrac{2 + (\gamma - 1)M_i^2}{\gamma + 1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}}{\dfrac{(\gamma + 1)M_i^2}{2 + (\gamma - 1)M_i^2} - 1} \tag{8}$$

The derivative $dM/d\dot{z}$ is indeterminate at $M = 1.0$ but can be determined by the L'Hospital's rule. In this case

$$\lim_{M \to 1.0} \frac{dM}{d\dot{z}} = \frac{\gamma + 1}{2} \tag{9}$$

Therefore, one should avoid the iteration when $\dot{z} = 0$. For such a case, however, the solution is known.

Transonic Flow around Symmetric Aerofoils at Zero Incidence

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Introduction

ONE of the earliest attempts to treat the problem of mixed flows at transonic speeds was the integral method of Spreiter and Alksne[1] based on the nonlinear transonic small disturbance equation. This technique was developed in 1954 before the days of high speed computors and consequently involves a minimum of numerical work. Results obtained for circular-arc aerofoils at zero incidence compare unfavourably with the result of the more recent numerical methods[2-4] particularly as regards the shock location. The unsatisfactory nature of the results of Ref. 1 are a direct consequence of approximations introduced into the fundamental integral equation in order to simplify the numerical work. In this note it is shown that the introduction of a simple correction factor, depending only on the transonic similarity parameter, improves the accuracy of the method of Ref. 1. It is suggested that this correction factor is a universal function of the transonic similarity parameter which, once established, can be used in subsequent calculations. The correction factor is found by using the results of the recent numerical methods[2,3] to locate the shock wave correctly in a number of examples.

Analysis

For a freestream Mach number M_∞ and a transonic parameter k the fundamental integral equation obtained by Spreiter and Alksne[1] for symmetric aerofoils at zero incidence is

$$\bar{u}(x,o) - \frac{\bar{u}^2(x,o)}{2} = \bar{u}_{T\mathfrak{L}}(x) + g(x,x_s) \tag{1}$$

where $\bar{U}(x,o)$ is related to the surface perturbation velocity in the freestream direction, $U(x,o)$, by

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